CBCS SCHEME

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15MAT11

First Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find y_n if $y = \dfrac{1}{x^2 - 5x + 6}$. **(06 Marks)**

b. Find the angle between the curves $r = a(1+\cos\theta)$ $r^2 = a^2 \cos 2\theta$ **(05 Marks)**

c. Find the radius of curvature for the curve $y^2 = \dfrac{4a^2(2a-x)}{x}$ where the curve meets x-axis. **(05 Marks)**

OR

2 a. If $x = \sin t$ $y = \cos mt$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$ **(06 Marks)**

b. Find the Pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ **(05 Marks)**

c. Show that for the curve $r(1-\cos\theta) = 2a$ ρ^2 varies as r^3. **(05 Marks)**

Module-2

3 a. Obtain the Taylor's expansion of $\tan^{-1}x$ in powers of $x - 1$ up to the term containing fourth degree. **(06 Marks)**

b. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x^2} - \cot^2 x\right)$. **(05 Marks)**

c. If $z = x^2 \tan^{-1}\left(\dfrac{y}{x}\right) - y^2 \tan^{-1}\left(\dfrac{x}{y}\right)$ show that $\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{x^2 - y^2}{x^2 + y^2}$. **(05 Marks)**

OR

4 a. Using Maclaurin's series prove that $\sqrt{1+\sin 2x} = 1 + x - \dfrac{x^2}{2} + \dfrac{x^4}{24}$ ...... **(06 Marks)**

b. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(05 Marks)**

c. If $u = \sqrt{x_1 x_2}$ $v = \sqrt{x_2 x_3}$ $w = \sqrt{x_3 x_1}$ find $J\left(\dfrac{u, v, w}{x_1 x_2 x_3}\right)$. **(05 Marks)**

Module-3

5 a. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ where t is the time. Find the velocity and acceleration at any time t and also their magnitudes at $t = 0$. **(05 Marks)**

b. Find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ **(05 Marks)**

c. Show that $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. Also find a scalar potential such that $\vec{F} = \nabla\phi$. **(06 Marks)**

OR

6 a. If $\vec{F} = (3x^2y - z)i + (xz^3 + y^4)j - 2x^3z^2k$ find grad (div $\vec{F}$) at (2, −1, 0) **(06 Marks)**

b. Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both solenoidal and irrotational. **(05 Marks)**

c. Prove curl (grad ϕ) = 0 for any scalar function ϕ. **(05 Marks)**

Module-4

7 a. Obtain reduction formula for $\int_0^{\pi/2} \sin^n x\,dx$ where n is a positive integer. **(06 Marks)**

b. Evaluate $\int_0^{\pi/6} \cos^4 3x \sin^2 6x\,dx$ using reduction formula. **(05 Marks)**

c. Solve $\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(05 Marks)**

OR

8 a. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$ where n is a positive integer. **(06 Marks)**

b. Obtain the orthogonal trajectory of the family of curves $r = a(1+\sin\theta)$ **(05 Marks)**

c. If the temperature of the air is 30°C and metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach temperature of 40°C . **(05 Marks)**

Module-5

9 a. Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. **(06 Marks)**

b. Solve by Gauss Jordan method $2x + 5y + 7z = 52$, $2x + y - z = 0$, $x + y + z = 9$. **(05 Marks)**

c. Find the largest eigen value and the corresponding eigen vector by power method given that $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ by taking the initial approximation to the eigen vector as $[1 \quad 0.8, -0.8]^1$. **(05 Marks)**

OR

10 a. Use Gauss seidel method to solve the equations
$x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$. **(06 Marks)**

b. Reduce the matrix to diagonal form $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ and hence find A^4. **(05 Marks)**

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form. **(05 Marks)**

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